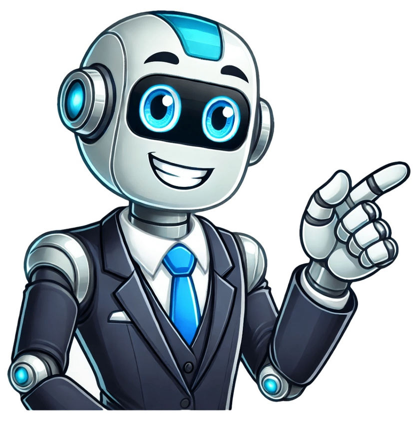

Hey everyone, i have a question here on 2D problems that I'm pretty much stuck on A ferry boat has a speed of 9.0km/h in calm water. Its pilot takes it on a heading due north across a river that has a current of 4.0km/h west. It takes 15 minutes to cross the river. a) how far downstream does the ferry land? (the answer is 1.0km) for a, (i've drawn a diagram) I've found the width of the river which is 2.3km and I've also found the velocity of the boat relative to the shore which is 9.8 km/h, 24 degrees W of N. from there, I'm stuck. please help! TY! Last edited: Apr 27, 2005 You don't need the widht of the river. All you need to know is that he was crossing for 15 minutes, and that the current was 4km west. This is enough to find out how far he drifted during the cross. oooo...hehehe, i feel sort of silly after just calculating that. thank you. but i have one more question, just wondering: why doesn't the boat's 9.0km/h affect how far the boat lands? wouldn't that impact the angle in which the boat is going across the river along with the 4.0km/h? i hope you understand what I'm trying to say... The ferryman maintains a strictly northward heading. Thus, his velocity North will compound with but will not affect his velocity West. The velocities are at 90 degrees to one another. You see? Think of it this way. West is the direction down the x-axis (left, and into the negative numbers), while North is the direction up the y-axis. This allows us to use vector notation. [tex] \vec v = [-4.0 \frac{km}{h}, 9.00 \frac{km}{h}][/tex] The x and y components (or i and j, as they're often called) are separate. Last edited by a moderator: Apr 27, 2005 YES! totally makes sense to me Thx for the help! jen Example: Boat has velocity 5m/s with respect to the river. It aims to reach point A, however, because of the river speed it reaches another point B. If the speed of current is 2m/s to the east calculate the time of trip and the distance between A and B. Boat follows the path shown in dashed line which is the direction of the resultant vector. This example can be examined under two part vertical and horizontal motion as in the case of projectile motion. Example: Velocity of the boat with respect to river is 10 m/s. It passes the river and reaches opposite shore at point C. If the velocity of the river is 3m/s, find the time of the trip and distance between B and C. Kinematics Exams and Solutions The boat and river problem is a classic example of relative motion, illustrating how the movement of a boat in a river is influenced by both the speed of the boat and the current of the river. This problem is highly applicable in real-life scenarios, such as navigating a boat across a river or planning the most efficient route in water bodies with strong currents. Understanding the dynamics between the boat's velocity and the river's current helps in determining the shortest path, the time taken to cross the river, or the boat's actual trajectory relative to the ground. This concept is crucial not only for sailors and river pilots but also in designing and planning transportation and logistics in riverine environments. By solving the boat and river problem, one can gain insights into optimizing travel time and fuel consumption, making it an essential topic in both practical navigation and theoretical physics.Boat River ProblemTo solve any riverboat problem, two things are to be kept in mind. A boat's speed with respect to the water is the same as its speed in still water. The velocity of the boat relative to water is equal to the difference in the velocities of the boat relative to the ground and the velocity of the water with respect to the ground.Now, start with the important term related to relative velocity.Important Terms $ \begin{aligned} d & =\text { width of river } \\ U & =\text { speed of river } \\ V & =\text { Speed of Boat w.r.t. River } \end{aligned} $ and $V_b=$ Speed of boat w.r.t. Ground So, the relation between u, v and V_b is $ V_b=U+V $$\text { Let's try to find out } V_b \text { in some important cases }$When the boat travels downstream (u and v have the same direction) Then, $V_b=(U+V) \hat{i}$When the boat travels upstream (u and v have opposite directions) Then,$V_b=(U-V) \hat{i}$$\text { If the boat travels at some angle } \theta \text { with river flow (u) }$ Now resolve v in two-componentComponent of v along $U=v_x=v \cos \theta \hat{i}$Component of v perpendicular to $U=v_y=v \sin \theta \hat{j}$So,$V_b=(v \cos \theta+u) \hat{i}+v \sin \theta \hat{j}$and, $\left|V_b\right|=\sqrt{u^2+v^2+2 u v \cos \theta}$Now if the time taken to cross the river is tThen,$t=\frac{d}{v \sin \theta}$Here x = driftAnd,$x=(u+v \cos \theta) t=\frac{(u+v \cos \theta) d}{v \sin \theta}$Important CasesNow, we will study some of the important cases which are mentioned below:To cross the river in the shortest timeThis means v is perpendicular to u$\begin{aligned} & \text { Or } \operatorname{Sin} \theta=1 \Rightarrow \theta=90^{\circ} \\ & \text { So. }\left|V_b\right|=\sqrt{u^2+v^2} \end{aligned}$Time taken $\quad t_{\min }=\frac{d}{v}$ Drift along river flow, $\quad x=d\left(\frac{u}{v}\right)$To cross the river in the shortest pathMeans drift = 0$\begin{gathered} x=(u+v \cos \theta) t=0 \Rightarrow \cos \theta=\frac{-u}{v} \\ \left|V_b\right|=\sqrt{v^2-u^2} \end{gathered}$The time taken to cross the river is: $t=\frac{d}{v \sin \theta}$ $ t=\frac{d}{\sqrt{v^2-u^2}}$Recommended Topic VideoSolved Example Based On Boat River ProblemExample 1: A man wishes to cross a river in a boat. If he crosses the river in a minimum time he takes 10 minutes with a drift of 120m. If he crosses the river taking the shortest route it takes 12.5 min. The velocity of the boat with respect to water is :1) $\frac{1}{3} m / s$ 2) $\frac{1}{4} \mathrm{~m} / \mathrm{s}$ 3) $\frac{1}{5} \mathrm{~m} / \mathrm{s}$ 4) $\frac{1}{6} m / s$Solution:For minimum time$t_1=\frac{d}{v}=10 \mathrm{~min}-(1)$$\begin{aligned} & \text { drift }=u \cdot t_1=120 \mathrm{~m} \\ & \text { or } u \cdot \frac{d}{v}=120 \Rightarrow u=\frac{120}{600} \mathrm{~m} / \mathrm{s} \Rightarrow u=\frac{1}{5} \mathrm{~m} / \mathrm{s} \end{aligned}$For Shortest route$\text { Let it make angle } \theta \text { with the river flow. }$$\begin{aligned} & \Rightarrow v \cos \theta=u \\ & \& t_2=\frac{d}{v \sin \theta}=\frac{d}{v \cdot \sqrt{1-\frac{u^2}{v^2}}}=\frac{d}{\sqrt{v^2-u^2}} \\ & \text { or } \frac{d}{\sqrt{v^2-u^2}}=12.5 \mathrm{~min}-(2) \end{aligned}$$\begin{aligned} & \text { Dividing Equation (2) to (1) } \\ & \frac{d}{v \cdot \frac{d}{\sqrt{v^2-u^2}}}=\frac{10}{12.5} \\ & \Rightarrow \sqrt{1-\frac{u^2}{v^2}}=\frac{4}{5} \\ & \Rightarrow 1-\frac{u^2}{v^2}=\frac{16}{25} \\ & \Rightarrow \frac{u^2}{v^2}=\frac{9}{25} \\ & \Rightarrow \frac{u}{v}=\frac{3}{5} \\ & \Rightarrow v=\frac{5}{3} \cdot u \\ & =\frac{5}{3} \times \frac{1}{5}=\frac{1}{3} \mathrm{~m} / \mathrm{s} \end{aligned}$ Hence, the answer is option (1).Example 2: A man can swim with a speed of 4km/hr in still water. He crosses a river 1 km wide that flows steadily at 3 kmph. If he makes his stroke normal to the river current, how far (in meters) down the river does he go when he reaches the other bank?1) 7502) 5003) 7004) 850Solution:Given- v= 4 km/hr, u=3 km/hr, D= 1km $\text { Drift }=\left(\frac{d}{v}\right) \cdot u=1 \mathrm{~km} \times \frac{3}{4}=750 \mathrm{~m}$Hence, the answer is option (1).Example 3: A person is swimming at a speed of 10 m/s at an angle of 120° with the flow and reaches a point directly opposite on the other side of the river. The speed of the flow is 'x' m/s. The value of 'x' to the nearest integer is ________.1) 502) 53) 204) 20Solution:To reach a point directly opposite on the other side of the river $ \begin{aligned} & V_{M / R} \sin 30^{\circ}=V_R \\ & 10 \sin 30^{\circ}=V_R \\ & V_R=x=5 \mathrm{~m} / \mathrm{s} \end{aligned} $Hence, the answer is option (5).Example 4: The swimmer crosses the river along the line making an angle of 45º with the direction of flow. The velocity of the river is 5 m/s. A swimmer takes 6 seconds to cross the river of width 60 m. The velocity of the swimmer with respect to water will be:1) $10 \mathrm{~m} / \mathrm{s}$ 2) $12 \mathrm{~m} / \mathrm{s}$ 3) $5 \sqrt{5} \mathrm{~m} / \mathrm{s}$ 4) $100 \mathrm{~m} / \mathrm{s}$Solution:River flow along y direction with a velocity of 5 j m/s. The swimmer crossover along the vector: i + j since the angle is 45 degree displacement of swimmer just after crossing = 60 i + 60 j meters speed of the swimmer with respect to a stationary observer = (60 i + 60 j) / 6 m/s = 10 i + 10 j m/s velocity of swimmer relative to the river = 10 i + 10 j - 5 j magnitude of velocity relative to river = 5√5 m/sHence, the answer is the Option (3).Example 5: A man who has a speed of 5km/h in still water crosses a river of width 1km along the shortest possible path in 15 minutes. The velocity of river water in km/h is :1) 32) 43) 84) 10Solution:$ \begin{aligned} & \cos \theta=\frac{4}{5} \\ & \sin \theta=\frac{3}{5} \end{aligned} $Now, $ V_R=5 \times \frac{3}{5}=3 \mathrm{Km} / \mathrm{h} $SummaryIn short, we have studied in depth about boat river problem along with different cases.. This is one of the most significant concepts in the kinematics chapter. In general, we encounter questions based on this principle in competitive exams like as NEET and JEE. Homework Statement (a) At what angle Sumon has to drive the boat to reach college at time? Context: The 4 km width river is situated in front of Sumon's house and his college is in opposite of that river. One day Sumon started his journey for college at 7.30 am by making an angle of 50° with a velocity of 5 km/h. College starts at 8.30. Velocity of current is 2 km/h. Relevant Equations The 4 km width river is situated in front of Sumon's house and his college is in opposite of that river. One day Sumon started his journey for college at 7.30 am by making an angle of 50° with a velocity of 5 km/h. College starts at 8.30. Velocity of current is 2 km/h. My answer is 130.54... Is that correct? Welcome to PF. Please describe how you calculated that answer, and show all of your work that led to it. Thanks. Oh, and 130 degrees with respect to what? Likes nasu and zetlearn Welcome to PF. Please describe how you calculated that answer, and show all of your work that led to it. Thanks. Oh, and 130 degrees with respect to what? Respect to Direction of Current Any idea how to solve it please?? Sorry, several things don't make sense. The problem statement seems to be giving you an angle of travel (with respect to a line that goes straight across the river, not pointing in the direction of the current). They give you a start time and the velocities, so it seems like they want to know when the boat reaches the other side (and probably want you to verify that the boat makes it straight across). So why are you trying to calculate an angle? Aren't you supposed to calculate the travel time? Sorry, several things don't make sense. The problem statement seems to be giving you an angle of travel (with respect to a line that goes straight across the river, not pointing in the direction of the current). They give you a start time and the velocities, so it seems like they want to know when the boat reaches the other side (and probably want you to verify that the boat makes it straight across). So why are you trying to calculate an angle? Aren't you supposed to calculate the travel time? The question is: (a) At what angle Sumon has to drive the boat to reach college at time? Then why do you say that it travels at 50 degrees? The text says: "by making an angle of 50°" . You need a good, coherent question before even thinking of what the answer may be. jbriggs444 The question is: (a) At what angle Sumon has to drive the boat to reach college at time? You seem to have interpreted the question as: "At what angle (relative to directly downstream) must Sumon drive so that he covers exactly four miles (not necessarily in the right direction) in one hour of travel time?" You correctly calculate that after one hour, the river will have carried Sumon 2 miles downstream. The boat will have travelled 5 miles relative to the water. We want the vector sum of that 2 mile vector and the 5 mile vector to have a magnitude of 4 miles. So you use the law of cosines and solve for the angle between the 2 mile vector (directly downstream) and the 5 mile vector (the direction taken by the boat). Let us take this to the next step. Where will Sumon be after one hour if he follows your 130 degree heading? Let us do it by components. We will use the ##x## axis for the direction of the current flow and the ##y## axis for the direction across the river toward the college. We will use ##w## for the distance travelled by the water and ##b## for the distance travelled by the boat relative to the water. We will use ##s## for the total distance. ##w_x = 2##, ##w_y = 0## ##b_x = 5 \cos 130 = -3.2##, ##b_y = 5 \sin 130 = 3.8## ##s_x = -1.2##, ##s_y = 3.8## So he has made it only 3.8 miles across the river and has ended up 1.2 miles upstream. Maybe he needs to aim more nearly directly across so that he actually hits the college instead of missing? Is it really a problem if he arrives early? Last edited: May 15, 2023 Description This is a simulation of a boat crossing a river. Adjust the direction the boat is aimed, the boat's velocity relative to the river, and the river's velocity relative to the earth. Press the "Run" button to watch the boat's trip across the river. Questions to answer: 1) What direction should the boat be aimed to get to the other side of the river in the least amount of time? 2) What direction does the boat need to be aimed to get to the point directly across the river? 3) If the boat is aimed directly across the river, does the speed of the river's current affect the amount of time it takes the boat to cross the river? Problem Statement: A river has a width d = 40 m. The water flows at a constant velocity vA = – 6 i (m/s) with respect to a frame of reference at rest O . A boat wants to cross the river from point A on one bank to point B on the other (see figure). The velocity vector of the boat with respect to the water is: v′B = 4 j (m/s). A cyclist is crossing the bridge at a constant velocity vC = 2 j (m/s) with respect to O. Calculate: The velocity vector of the boat with respect to O. The velocity vector of the boat with respect to the cyclist. The time it takes for the boat to cross the river. The distance between points A and B. If the current had twice the speed, how long would it take for the boat to cross the river? Solution: In this problem we have velocities measured with respect to two different frames of reference. The first one (O) is at rest and the second one, the water, which we will call O′, moves at constant velocity with respect to O. Since both are inertial, we will have to make use of the Galilean transformations. More specifically, the Galilean transformation for the velocity is given by: Where v is the velocity with respect to a frame of reference at rest and v′ is measured with respect to the frame of reference moving at constant velocity with respect to the first one. Therefore in the equation above the velocity of the boat with respect to O will be v and v′ will be the velocity of the boat with respect to the water (v′B in the problem statement). V is the velocity of the moving frame of reference with respect to the reference frame at rest (in this case the velocity of the water with respect to O, vA). If we apply the Galilean transformation for the velocities to this problem we have: And substituting the corresponding numerical values: This page contains affiliate links and I earn a commission if you make a purchase through one of the links, at no cost to you. As an Amazon Associate I earn from qualifying purchases. This does not affect the information provided on the pages of this website. Product prices and availability displayed on this page are updated multiple times daily. They are accurate as of the date/time indicated and are subject to change. Any price and availability information displayed on Amazon.de at the time of purchase will apply to the purchase of the products. In order to calculate the velocity of the boat with respect to the cyclist we use the Galilean transformation for the velocities, but in this case the frame of reference in motion O′ will be the cyclist. Therefore the Galilean transformation is now given by: Substituting the givens, the velocity of the boat with respect to the cyclist is: In order to calculate the time it takes for the boat to cross the river we use its width d and the vertical component of the velocity of the boat with respect to O. Since the last one is constant, the relationship between the space traveled by the boat in a straight line between the two banks, its speed on that axis and the time it takes is: We can use this time to calculate the horizontal distance that the boat has traveled in the same amount of time. We use the horizontal component of its velocity: And using the Pitagoras' theorem we determine the distance between A and B: If the current had twice the speed, it would take the same time for the boat to cross the river, since the vertical component of its velocity would be the same. However, it would travel a greater distance in the same amount of time. The post Relative Motion - Riverboat problem appeared first on YouPhysics River-boat problems are a classic category of kinematics questions that involve calculating the motion of a boat as it travels across a river with a current. These problems typically require an understanding of vectors, as the boat's velocity and the river's current velocity must be combined to find the resultant velocity, which determines the actual path of the boat. Understanding the Basics Before diving into the specifics of river-boat problems, it's important to understand some basic concepts: Velocity: A vector quantity that includes both speed and direction. Current: The flow of the river, which has its own velocity vector. Resultant Velocity: The vector sum of the boat's velocity and the current's velocity. Key Formulas In river-boat problems, we often use the following formulas: Resultant Velocity: $\vec{V}_r = \vec{V}_b + \vec{V}_c$ $\vec{V}_r$ is the resultant velocity. $\vec{V}_b$ is the boat's velocity relative to the water. $\vec{V}_c$ is the current's velocity. Time to Cross the River: $t = \frac{d}{V_{b_y}}$ t is the time to cross. d is the width of the river. V_{b_y} is the component of the boat's velocity perpendicular to the current. Downstream/Upstream Distance: $d_{\text{stream}} = V_{c_x} \cdot t$ d_{stream} is the distance traveled downstream or upstream due to the current. V_{c_x} is the component of the current's velocity along the river. t is the time to cross the river. Table of Differences and Important Points Aspect Description Direction of Travel The boat can attempt to travel directly across the river, upstream, or downstream. Boat's Velocity The velocity of the boat relative to the water, not affected by the current. Current's Velocity The velocity of the water itself, which moves the boat downstream if not compensated for. Resultant Velocity The actual path the boat takes, considering both the boat's velocity and the current's velocity. Crossing Time The time it takes for the boat to reach the opposite side of the river. Drift The sideways movement of the boat caused by the current, also known as the downstream distance. Examples Example 1: Crossing Perpendicular to the Current A boat wants to cross a river that is 300 meters wide. The boat can move at 5 m/s relative to the water, and the current flows at 3 m/s. How long does it take to cross the river, and how far downstream will the boat be carried by the current? Solution: Calculate the time to cross the river: $$ t = \frac{d}{V_{b_y}} = \frac{300 \text{ m}}{5 \text{ m/s}} = 60 \text{ s} $$ Calculate the downstream distance: $$ d_{\text{stream}} = V_{c_x} \cdot t = 3 \text{ m/s} \cdot 60 \text{ s} = 180 \text{ m} $$ The boat will take 60 seconds to cross the river and will be carried 180 meters downstream. Example 2: Crossing with No Downstream Drift A boat wants to cross the same river without any downstream drift. What angle must the boat head, and what is the crossing time? Solution: The boat must head upstream at an angle θ such that the upstream component of its velocity cancels out the current's velocity. Using trigonometry, we find: $$ \tan(\theta) = \frac{V_{c_x}}{V_{b_y}} = \frac{3}{5} $$ $$ \theta = \arctan\left(\frac{3}{5}\right) \approx 31^\circ $$ The crossing time remains the same as in Example 1, since the perpendicular component of the boat's velocity is unchanged: $$ t = 60 \text{ s} $$ The boat must head approximately 31° upstream to have no downstream drift and will still take 60 seconds to cross the river. River-boat problems can vary in complexity, but by breaking them down into their vector components and applying the principles of kinematics, they become manageable. Understanding how to resolve vectors and apply trigonometry is essential to solving these problems effectively. "It takes the same amount of time for the boat to travel to point C to D as it takes the hat to travel from point B to point D." Surely that's not true. The hat didn't sit at point B waiting for 20 minutes until the boat reached point C and then start floating downstream as soon as the boat turned back upstream! What is true is that the time taken for the boat to go down stream from B to C and then back from C to D is the same as the time taken for the hat to float from B to D. The boat and hat were together at point B and then again at point D. The time elapsed is the same for both. Call the speed of the boat, relative to the water, u and the current speed s. Since the boat is initially traveling down stream, its speed, relative to the bank, is u+ v. When it turns and goes back up stream, its speed, relative to the bank, is u- v. Of course, the speed of the hat, relative to the bank, once it has fallen out of the boat is the same as the speed of the water: v. When the hat falls out of the boat, the boat continues downstream for an unknown time t1 at speed u+v to point C: it travels a distance (u+v)t1. It then turns around and travels upstream for another unknown time t2 at speed u-v to point D: it travels a distance (u-v)t2. The actual distance from point B to point D is (u+v)t1- (u-v)t2= u(t1- t2)+ v(t1+ t2). During that time, the hat is floating downstream at speed v for time t1+ t2. The distance it floats, which is also the distance from B to D is v(t1+ t2. Setting those two different calculations for the distance from B to D equal: u(t1- t2)+ v(t1+ t2)= v(t1+ t2. Notice that the "v(t1+ t2)" terms on each side cancel leaving us with u(t1- t2)= 0 or t1= t2. The time going downstream must be equal to the time going upstream! We can now note that t1 is given as 20 minutes so t2 is also 20 minutes. Unfortunately, we still have no way to find either u or v separately. The information that "The distance between point A and B is 1 mile" doesn't help at all since that has nothing to do with the hat. If we were told "the distance between point B and point D" is 1 mile, then, since we know it took the boat 20 minutes to go from point B to point C, and that is the same as the time the to go back up from C to D, we would know that the hat was floating for 40 minutes. We could then calculate that v= 1 mile/40 minutes= 1.5 miles per hour. I was hoping to explore the Calculus of Variations. How do we prove by Calculus of Variations that the minimum time for boat crossing a river (perpendicular to the current for starters) with current ##v_r##, and boat velocity in still water ##v_b## that the path will be a straight line? I tried, but I got lost. Let ##x## be the distance from launch point A to point B (directly opposite A) on the other bank. ## \rightarrow^{x^+} \downarrow^{y^+}## I get the following equations: $$ \frac{dx}{dt} = v_b \cos \theta \tag{1}$$ $$ \frac{dy}{dt} = v_b \sin \theta - v_r \tag{2}$$ I believe we are supposed to minimize the following: $$ T = \frac{1}{v_b} \int \frac{dx}{ \cos \theta} $$ I notice that using the Chain Rule on ##(2)##: $$ \frac{dy}{dx} = \tan \theta - \frac{v_r}{v_b} \sec \theta $$ Thats implies that: $$ 0 = \int dy = \int \left(\tan \theta - \frac{v_r}{v_b} \sec \theta \right) dx $$ Is it then not only the case that the integral on the RHS is zero if: $$ \tan \theta - \frac{v_r}{v_b} \sec \theta = 0 \implies \sin \theta = \frac{v_r}{v_b} $$ Which means that ## \theta ## is a constant... i.e. the path is a straight line. I realize this doesn't show the time is minimized though. What is the proper way to go about proving ##(1)## is minimized using the Calculus of Variations? kuruman I was hoping to explore the Calculus of Variations. How do we prove by Calculus of Variations that the minimum time for boat crossing a river (perpendicular to the current for starters) with current ##v_r##, and boat velocity in still water ##v_b## that the path will be a straight line? The path is always a straight line because the acceleration is zero. If you are thinking about the angle at which the boat must be aimed, clearly the time is ##T=\frac{L}{v_0\cos\theta}##. Using calculus of variations to find the minimum of that is like using a sledgehammer to kill a flea. Perhaps you might wish to explore the following situation. A lifeguard can run on sand at top speed ##v_s## and swim in water at top speed ##v_w## (##v_w